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Observations On The Surd Equation

$$\sqrt{2z-4}=\sqrt{x+\sqrt{(m^2+k)y}}+\sqrt{x-\sqrt{(m^2+k)y}}\ (m\neq 0)$$

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Abstract:

In this paper, non-zero integer solutions to the surd equation with three unknowns given by $\sqrt{2z-4}=\sqrt{x+\sqrt{(m^2+k)y}}+\sqrt{x-\sqrt{(m^2+k)y}}$ are obtained.

Keywords: surd equation, transcendental equation ,integer solutions

Introduction:

Diophantine equations have an unlimited field of research by reason of their variety. Most of the Diophantine problems are algebraic equations [1,2]. In [3-18], the integral solutions of transcendental equations involving surds are analyzed for their respective integer solutions.

This communication analyses a transcendental equation with three unknowns given by $\sqrt{2z-4}=\sqrt{x+\sqrt{(m^2+k)y}}+\sqrt{x-\sqrt{(m^2+k)y}}$. Infinitely many non-zero integer triples (x,y,z) satisfying the above equation are obtained.

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Notations:

$$t_{3,n} = \frac{n(n+1)}{2}$$

$$CP_n^4 = \frac{n(4n^2+2)}{6}$$

$$CP_n^{12} = \frac{n(12n^2-6)}{6}$$

Method of analysis:

The surd equation to be solved is

$$\sqrt{2z-4} = \sqrt{x+\sqrt{(m^2+k)y}} + \sqrt{x-\sqrt{(m^2+k)y}} \ (m \neq 0) \qquad (1)$$

On squaring both sides of (1),it simplifies to

$$z = 2 + x + \sqrt{x^2-(m^2+k)y^2} \qquad (2)$$

To start with ,observe that the square root on the R.H.S. of (2) is removed

by choosing

$$x = s(2m^2+k) \quad , y = 2sm \quad k \geq 0 \qquad (3)$$

and from (2)

$$z = 2s(m^2+k)+2 \qquad (4)$$

A few numerical solutions are presented in Table:1 below

Table:1 Numerical solutions

s	k	m	x	y	z
1	1	1	3	2	6
1	2	1	4	2	8
2	3	1	10	4	18
2	2	3	40	12	46
4	3	5	212	40	226

Observations :

1. $m(2z-2x-4)=ky$

2. $(z-my-1)^2=1+8t_{3,ks}$

3. $y+x=sk+4st_{3,m}$

4. $\frac{3(2mx-ky)}{y}$ is a nasty number

5. $y(m+1)=4st_{3,m}$

6. $y(x-(k-1)s)=6s^2CP_m^4$

7. $7x-6y=s(7k+2t_{16,m})$

8. $(7m-6)y=2st_{16,m}$

9. $(14m-13)y=2st_{30,m}$

10. $3y(x-s(k+1))=6s^2CP_m^{12}$

However , there are other choices of x ,y for eliminating the square-root on the R.H.S. of (2).

The corresponding values of x ,y along with z are exhibited in Table:2 below:

Table: 2 Choices of x ,y ,z

x	y	z
$(m^2+k+1)s$	2s	$2((m^2+k)s+1)$
$(m^2+k)p^2+q^2$	$2pq$	$2((m^2+k)p^2+1)$
$2m^{s+2}+km^s$	$2m^{s+1}$	$2m^{s+2}+2km^s+2$

Conclusion:

In this paper, we have presented integer solutions to the surd equation

$$\sqrt{2z-4}=\sqrt{x+\sqrt{(m^2+k)y}}+\sqrt{x-\sqrt{(m^2+k)y}}\ (m\neq 0)$$

. To conclude one may attempt to find integer solutions to other choices of surd equations .

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